Clairaut’s theorem

Alias

Schwarz’s theorem

Clairaut’s theorem

Young’s theorem

Intro

It says that the symmetric of derivatives for multiple variables is true.

Statement

Given a vector of variables = and a function (i.e. the input is a vector type while the ouput is too.)

There is a property, the symmetric of derivatives for multiple variables. More exactly to say, they will return same result if the order of derivatives is changed. That is,

=

It can be generalized into derivatives many times. That is, for example,

=

Proof of statement

Proof by induction

Suppose:

Suppose refers the statement is valid and true with given a vector of different variables. Represent with mathematical way, it always holds:

: =

where

=

Base step:

For = 1, the following always holds. (It is true and one don’t have to prove it.)

: With given = ,

=

For = ,

: With given = ,

=

To prove it, first use the mean value theorem.

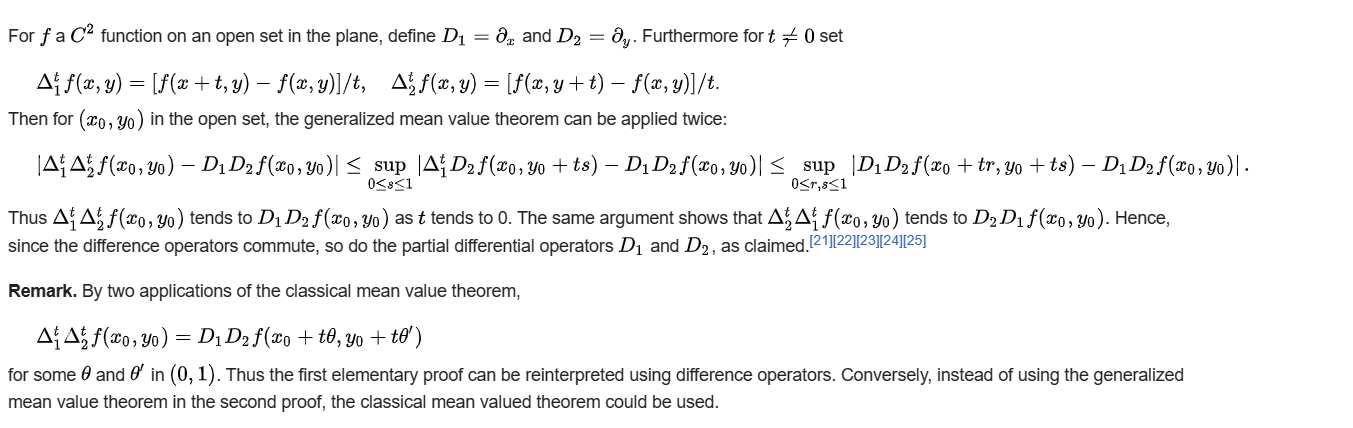
Given a function such that is continuous at (of course ) and differentiable at interval , then there exist such that

=

Since

,

the equality holds.



Hence, the proof of base step is completed.

Inductive step:

By inductive hypothesis, is true

: With given vector = ,

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One has to prove for is also true.

: With given vector =

=

=

(by inductive hypothesis)

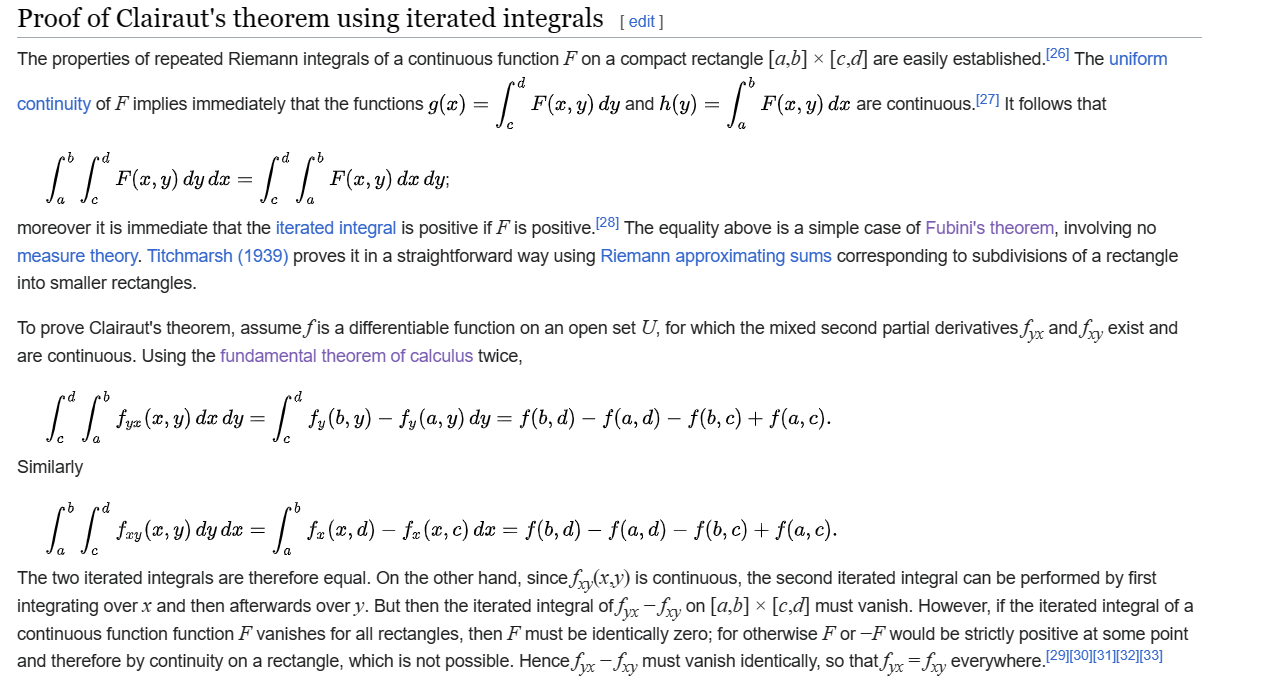
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(by base step)

which completes the proof.

Method 2:

By iterated integrals.



Ref

For a shorter but less detailed about proof of Clairaut’s theorem, see

[Symmetry of second derivatives - Wikipedia](https://en.wikipedia.org/wiki/Symmetry_of_second_derivatives#Schwarz's_theorem)

For a more detailed but longer proof of it, see

[Wayback Machine (archive.org)](https://web.archive.org/web/20060518134739/http://are.berkeley.edu/courses/ARE210/fall2005/lecture_notes/Young%27s-Theorem.pdf)